channels and the reclamation of land in consequence have resulted. It is in this way that the lake occupying the depression of the Fayum has been diminished in area.

The floods in the *Nile Delta* of which records have been kept, that are trustworthy for the past 175 years at least, have been critically studied by Captain Lyons with the view of discovering the determining causes of their variations. While no regular periodic alternations of high and low floods can be detected by the study of these records, their dependence on the rainfall and the distribution of atmospheric pressure in the highlands of Equatorial Africa is very apparent. There is reason to believe that more numerous, systematic, and complete meteorological observations in the districts outside Egypt may enable us, in the end, to predict from month to month the probable fluctuations of the annual Nile flood.

The space at our command has only permitted the notice of a few of the more salient features of this very interesting volume. In conclusion, we must congratulate Captain Lyons and the Egyptian Government upon the great amount of valuable work which has been accomplished and is still in progress. A word of praise must also be added on the excellent typography of the volume, and the admirable plates with which it is illustrated.

J. W. J.

THE HISTORY OF DETERMINANTS.

The Theory of Determinants in the Historical Order of Development. Part i. Second edition. General Determinants up to 1841. Part ii. Special Determinants up to 1841. By Dr. T. Muir, C.M.G., F.R.S. Pp. xii+492. (London: Macmillan and Co., Ltd., 1906.) Price 17*s.* net.

A MATHEMATICAL history of the right sort is much more than a mere bibliography, and in some respects is more valuable than a treatise on the subject with which it deals. It helps us to see how mathematical ideas originate, and how, as they become familiar, the symbolism by which they are expressed becomes compact and appropriate. This is especially the case with determinants, because a determinant is essentially a comprehensive symbol, and it would perhaps be more proper to speak of the calculus than of the theory of determinants. It may seem strange, at first sight, to find a history so large as this dealing with a subject so limited; but no one can complain that the author is either diffuse or irrelevant, and his work may be praised without restriction as a model of its kind.

It is unnecessary to say much of the first part, which is mainly a reprint of the volume which appeared in 1900. Dr. Muir has written a new introduction, and added a few additional notices. Two things cannot fail to strike the reader of this part. The first is the great supremacy of Cauchy and Jacobi in everything relating to choice of notation and clearness of statement; the other is the great and long unrecognised ability of Schweins. Schweins, in a way, brought this fate upon himself; his style is heavy, and his notation cumbrous in the extreme, but his contributions to the subject are of great value and generality, although they attracted no notice for many years, and were re-discovered by others. Unfortunately, they are expressed in such a repulsive notation that no one but an enthusiast would read his works, and the student will feel very grateful to Dr. Muir for his analysis of them. Part of this analysis, in some ways the most interesting, is given on pp. 311–322; this, and the subsequent section on a paper of Sylvester's, deserve careful reading, because, as Dr. Muir points out, Schweins gives some results on alternants which even now are not familiar, and Sylvester makes some hasty statements which, as they stand, appear to be incorrect, but which, if corrected, or rightly interpreted, might lead to important formulæ.

It should be noted that on p. 323 the determinant is misprinted, a, a^2, &c., being put for a_1, a_2, &c. Moreover, it is not explained so clearly as it should be that $\zeta a^r = a_r$; while the law $a_r \,.\, a_s = a_{r+s}$ is *not* used. The right statement is $\zeta(a^r \,.\, a^s) = \zeta(a^{r+s}) = a_{r+s}$; while $\zeta(a^r)\zeta(a^s) = a_r a_s$. Readers of Sylvester's papers must be careful to distinguish this ζ from the square of the operator $\zeta^{\frac{1}{2}}$. It may be noticed, in passing, that these generalised alternants present themselves in the theory of numbers, both when the elements are roots of unity and also when they are not, so that further knowledge of their properties is desirable, and the suggestion made (p. 325) that Sylvester's results are true when the elements are periodic deserves further examination.

Considerable space is given to functional and orthogonal determinants, and here, of course, Jacobi receives most attention. The results are now so familiar that it requires some effort of imagination to realise the gain in working power which has resulted from Jacobi's investigations. In this connection attention may be directed to an odd remark on p. 297. Speaking of one of Jacobi's papers, Dr. Muir says:—"The only thing worth noticing is the curious cubic equation . . ."; this "curious" equation is nothing more nor less than the reducing cubic for two ternary quadratic forms, in the exact notation of Salmon's "Conics"! And Dr. Muir even takes the trouble to express the invariants Θ, Θ' in the forms

$$Aa' + Bb' + \ldots, \text{ \&c.,} \qquad A'a + B'b + \ldots, \text{ \&c.,}$$

as if this were a quite novel idea.

Returning for a moment to alternants and their applications, attention may be directed to the work of Jacobi and Cauchy on the expansions of rational functions of several variables (pp. 331–345). This is important in the theory of functions, in that of algebraic forms, and in that of partitions. In some ways it deserves further investigation; in various applications the expansions have to be infinite series, and the question of convergency has to be faced, even when the series are used for establishing formal equivalences; this is a curious case of formal and arithmetical algebra each marching, so to speak, on the other's domain.

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There is one more observation made by Dr. Muir (p. 290) which is rather puzzling. After giving some identities of Lagrange's, which are, in fact, relations between determinants, Dr. Muir says, "a reference to the original papers, already described, will make it almost perfectly certain that Lagrange did not view them in this light. The like is true of Gauss. . . ." Now Gauss, at any rate, used the term "determinant"; if this word is used in the modern sense of the *symbol*, of course Dr. Muir's remark is correct, but is then quite trivial; on the other hand, if it means the function, it is hard to see how Gauss, not to say Lagrange, could fail to see that their expressions involved determinants, especially as each was quite familiar with them in connection with the theory of numbers. This is particularly true of Gauss, who gives the name "determinant" to $(\alpha\beta\gamma)$ as well as to $(\alpha\beta)$.

It is a matter of regret that, although a bibliography of orthogonants (to 1840) has been given, Dr. Muir has not been able to include in this volume his valuable lists of writings relating to determinants. To have added them would not have increased the size of the book very much, and it would have been very convenient to have them here. But perhaps the author intends to give us the history of his favourite subject subsequent to 1841, the date at which he has now closed.

G. B. M.

EUROPEAN VERTEBRATES.

Die wirbelthiere Europa's mit Berücksichtigung der Faunen von Vorderasien und Nordafrika. By Dr. O. Schmiedeknecht. Pp. vi+470. (Jena: Gustav Fischer, 1906.) Price 10 marks.

IT is always convenient to have within covers separated by a moderate distance only an account of the fauna of a definite district, especially when, as in the present instance, the fauna is one that is fairly exhaustively known. It is not likely that a manual of the scope of that which we review here will ever need substantial alteration, or even slight changes, for many years to come. The volume, in fact, is not only of permanent value, but contains the marrow of a whole library of faunistic works, and includes all that the student needs, whether the aim of his studies be purely geographical or whether he desires a handy series of definitions of families, genera, and species. Inasmuch as the volume is something less than five hundred pages in length, and seeing that the definitions of family and other characters are often from twelve to twenty lines in length, the author is compelled to deny himself any discussion of points round which opinions fluctuate, and is driven to be entirely dogmatic.

It is therefore not everyone who will follow Dr. Schmiedeknecht with complete agreement from beginning to end. He will not, for example, please all of us by placing the "reed pheasant," *Panurus biarmicus*, among the tits, though it *is* frequently called the bearded tit; nor can we agree to the use of four separate generic names for the four species of rorquals, which appears to us as a recrudescence of one of the very worst achievements of the systematists of the past. In adopting an old scheme of arrangement for birds, the author is compelled thereby to separate widely the gulls and Limicoline birds, which many anatomists have concurred in placing in very close relationship. There are plenty of similar examples to be gathered from Dr. Schmiedeknecht's pages. If the author errs at all in the number of species which he admits into his manual, it is rather on the side of economy than profusion. Of the very long series of "species" of mice admitted, or rather insisted upon, by some British naturalists, Dr. Schmiedeknecht will only consider four as established. Perhaps he carries this plan a little too far in declining to admit the "Irish weasel," *Mustela hibernica*, which is not in any way referred to. Apropos of weasels, it will certainly surprise some persons to learn that the proper name of the common weasel is not *Mustela vulgaris*, but *M. nivalis*, inasmuch as (according to Dr. Schmiedeknecht) Linnæus gave the name to an individual in winter dress.

That the weasel, like its very close ally the stoat, changes to white in winter can hardly be a fact of general knowledge, since it is not mentioned in at any rate one well-known work upon British mammals.

In classifying the snakes, Dr. Schmiedeknecht follows a somewhat curious plan. He divides the European species into five families of equal rank, which are (in the order treated by him) Crotalidæ, Viperidæ, Colubridæ, Peropodidæ, and Typhlopidæ. To give the "pit vipers" a place in the system which divides them as far from the more typical Viperidæ as from the Peropodidæ, or Boidæ as most would prefer to call them, is quite opposed to the minute details of anatomical agreement between all vipers.

To such criticisms, however, the author might well reply that his arrangement is rather a sorting than a classification, and that, as a matter of fact, judged by external characters only (and it is these alone that are made use of), the two divisions of the vipers are very distinct, and the gulls are remote from the sandpipers, plovers, &c.

In his preface, Dr. Schmiedeknecht puts himself forward as a champion of the systematic aspect of zoology as a desirable commencement for the student of that science There is no doubt that most of us, in this country at least, were led to pursue zoological studies by reason of the fervour and enthusiasm engendered by the joys of collecting objects of natural history. The quality of knowledge possessed by the pure systematist of mature years is often but little in advance of this stage, and in remarking that "he must be as a rule a remarkable systematist who is not at the same time a biologist," the author is expressing an opinion which the annals of museums do not confirm. In fact, Dr. Schmiedeknecht's introductory remarks read a little like an apology, which is not at all needed in introducing so useful and accurate a work as that which we notice here.

F. E. B.

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